Design of LDPC Codes for Cooperative Diversity Systems

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Outline

• Background
• LDPC code design for cooperative relay systems
• Simulation results
• Conclusions
Background: Cooperative Relay System

- Relay node operations:
  - receive the signal from $S$;
  - amplify or decode the signal;
  - transmit the signal to $D$.
- Destination node operations:
  - receive and decode the signals from both $S$ and $R$.

(a) diagram of a single-relay system  
(b) one-dim channel model
Background: Two Relay Protocols

Simple protocol

\[ y_D(1) = h_{SD} w_1 + n_D \]
\[ y_D(2) = h_{SD} w_2 + h_{RD} w_1 + n_D \]

"Decode-and-forward" (DF) protocol

\[ y_D(1) = h_{SD} w_1 + n_D \]
\[ y_D(2) = h_{SD} w_1 + h_{SD} w_2 + h_{RD} w_1 + n_D \]
Background: Factor Graph (FG)

- Factor graph representation:
  entire FG (B-block) and partial FGs (PFG, 2-block)
- Optimum code design: over the entire FG
  high computational complexity and long delay in decoding
Background: FG Decoupling

- Suboptimum code design:
  over the PFG (instead of the entire FG)

- Successive decoding:
  - corresponding to the structure of successive PFGs
  - forward-decoding (FW) and backward-decoding (BW)
Code Design: Relay Operations

- Simple protocol: AWGN \( y_R(t) = \sqrt{P_S}h_{SR}w_t + n_R \)
- DF protocol: virtual MISO model

\[
y_R(t) = \sqrt{P_{S,1}}h_{SR}w_t + \sqrt{P_{S,2}}h_{SR}w_{t+1} + n_R
\]
i.e., \( y_R(t) = h_{SR}[\sqrt{P_{S,1}}, \sqrt{P_{S,2}}][w_t, w_{t+1}]^T + n_R \)

- EXIT chart analysis for LDPC-coded relay (virtual MISO)

\[
I_R(t) \text{ and } \sigma_R(t) = J^{-1}(I_R(t))
\]

- Two independent LDPC decoders: \( w_t \) and \( w_{t+1} \) (or \( w_{t-1} \))
- The relay output is the decoder output for \( \hat{w}_t \)
- The decoding of \( w_{t+1} \) only helps to improve the prior in the decoding of \( \hat{w}_t \)
- The relay output \( I_R(t) \) will be exploited as the prior during the time slot \( (t+1) \) when decoding \( \hat{w}_{t+1} \)
Code Design: Relay Performance Analysis

• Iterative receiver for the LDPC-coded relay node
Code Design: Destination Operations (Virtual MIMO Model)

\[ y(t) = Hx(t) + n(t), \]
\[ y(t) = [y_D(t), y_D(t + 1)]^T, \quad x(t) = [w(t), \hat{w}(t), w(t + 1), \hat{w}(t + 1)]^T \]

- **Simple protocol: MIMO**
  - FW-dec.:  
    \[ H_s^f = \begin{bmatrix} h_{SD}\sqrt{P_S} & 0 & 0 & 0 \\ 0 & h_{RD}\sqrt{P_R} & 0 & h_{SD}\sqrt{P_S} \end{bmatrix} \]
  - BW-dec.:  
    \[ H_s^b = \begin{bmatrix} 0 & h_{RD}\sqrt{P_R} & 0 & 0 \\ h_{SD}\sqrt{P_S} & 0 & 0 & h_{RD}\sqrt{P_R} \end{bmatrix} \]

- **DF protocol: MIMO**
  - FW-dec.:  
    \[ H_{DF}^f = \begin{bmatrix} h_{SD}\sqrt{P_{S,1}} & 0 & 0 & 0 \\ h_{SD}\sqrt{P_{S,2}} & h_{RD}\sqrt{P_R} & h_{SD}\sqrt{P_{S,1}} & 0 \end{bmatrix} \]
  - BW-dec.:  
    \[ H_{DF}^b = \begin{bmatrix} h_{SD}\sqrt{P_{S,2}} & h_{RD}\sqrt{P_R} & 0 & 0 \\ h_{SD}\sqrt{P_{S,1}} & 0 & h_{SD}\sqrt{P_{S,2}} & h_{RD}\sqrt{P_R} \end{bmatrix} \]
Code Design: Destination Performance Analysis

- **EXIT analysis for the LDPC-coded destination**

\[ I_D(t) = \sum_i \lambda_i J(\sqrt{[J^{-1}(I_{E,\text{Det}(i))}]^2 + i[J^{-1}(I_{E,\sqrt{ND}})]^2}) \]

- Two independent LDPC decoders: \( w_t \) and \( w_{t+1} \) (or \( w_{t-1} \))
- The destination output is the decoder output for \( \hat{w}_t \)
- The decoding of \( w_{t+1} \) only helps to improve the prior in the decoding of \( \hat{w}_t \)
- The relay output \( I_D(t) \) will be exploited as the prior in the PFG-(t+1) when decoding \( \hat{w}_{t+1} \)

Similar receiver structure as that for the relay
Code Design: Conventional Code Performance Analysis

- Relay decoding to get $\hat{w}_t$ for each $w_t$

The imperfect relaying effects are **numerically** incorporated into the destination performance via the virtual MIMO model.

$$y = Hx + n \text{ where } x = [w_t, \hat{w}_t, w_{t+1}, \hat{w}_{t+1}]$$

- The destination detector output $I_{E,Det}(i)$ within each iteration

$$\text{ML detection : } L_{E,Det} = \log \frac{\sum_{w_{t+1}} \sum_{\hat{w}_t} p(y|w_{t}=1)}{\sum_{\hat{w}_{t+1}} \sum_{\hat{w}_t} p(y|w_{t}=-1)}$$

**Numerically** obtained function: $I_{E,Det}(i) = f_{Det}(I_{A,Det}(i), \frac{E_b}{N_0})$

All **numerical** procedures, high computational complexity and thus slow evolution
Code Design: Semi-analytical Performance Analysis Approaches

• BSC approximation for the relay:
  \[ \hat{w}_t = f_R(w_t) = w_t \text{ or } -w_t \]
  with the crossover probability \( P_0 = P(\hat{w}_l \neq w_l | w_l) = Q\left(\frac{J^{-1}(I_R)}{2}\right) \)
  – Approximate the relay output using the BSC output \( \hat{w}_t \)
    (instead of numerically decoding to get \( \hat{w}_t \))

• Efficient destination detector with Gaussian approx.:
  – Computationally efficient detection (instead of ML)
    Semi-analytical expressions are available for \( L_{E,Det} \).
  – \( I_{E,Det}(i) \) also has semi-analytical expressions including the parameter \( P_0 \) (instead of numerically calculation)
  – The imperfect relaying effects are analytically incorporated into the destination performance via \( P_0 \)
## Results: Destination Performances (1)

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<tbody>
<tr>
<td><strong>Simple</strong></td>
<td>0.25</td>
<td>FW</td>
<td>-1.25dB</td>
<td>-0.30dB</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>BW</td>
<td>-1.55dB</td>
<td>-0.75dB</td>
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<tr>
<td></td>
<td>0.50</td>
<td>FW</td>
<td>-2.50dB</td>
<td>-1.50dB</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>BW</td>
<td>-2.90dB</td>
<td>-2.10dB</td>
<td></td>
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<tr>
<td><strong>DF</strong></td>
<td>0.25</td>
<td>FW</td>
<td>-3.50dB</td>
<td>-3.45dB</td>
<td>-2.61dB</td>
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<td></td>
<td></td>
<td>BW</td>
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<tr>
<td></td>
<td>0.50</td>
<td>FW</td>
<td>-3.70dB</td>
<td>-3.25dB</td>
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<tr>
<td></td>
<td></td>
<td>BW</td>
<td>-3.70dB</td>
<td>-3.26dB</td>
<td>-2.37dB</td>
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</table>
Results: Destination Performances (2)

- **Significant gain over regular codes:**
  - Optimized codes versus regular codes (0.8dB gain)

- **Capacity-approaching performance:**
  - Optimized codes versus capacity (within 0.1dB gap)
    - (DF protocol, d=0.25)

- **Successive decoding schemes:**
  - BW-dec. outperforms FW-dec. (0.3~0.4dB gain)
    - BW versus FW (simple protocol)
  - BW-dec. has approximated perf. as FW-dec.
    - BW versus FW (DF protocol)

*Imperfect relaying effects!*
### Results: Relay Performances (1)

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<tbody>
<tr>
<td>Relay simple</td>
<td>0.25</td>
<td>-1.25dB (FW)</td>
<td>-1.55dB (BW)</td>
<td>-4.98dB</td>
<td>-4.17dB</td>
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<tr>
<td>Relay simple</td>
<td>0.50</td>
<td>-2.50dB (FW)</td>
<td>-2.90dB (BW)</td>
<td>-3.26dB</td>
<td>-2.40dB</td>
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<tr>
<td>DF</td>
<td>0.25</td>
<td>-3.45dB (FW)</td>
<td>-3.47dB (BW)</td>
<td>-3.45dB</td>
<td>-2.62dB</td>
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<tr>
<td>DF</td>
<td>0.50</td>
<td>-3.25dB (FW)</td>
<td>-3.26dB (BW)</td>
<td>-3.23dB</td>
<td>-2.35dB</td>
</tr>
</tbody>
</table>
Results: Relay Performances (2)

• Significant gain over regular codes
  – Optimized codes versus regular codes (0.8dB gain)
  – Note: The optimized codes are designed for the destination (instead of the relay).

• Perfect relaying
  – Relay perf. versus Dest. perf. (simple protocol)
  – Destination perf.: \textit{BW outperforms FW} (over 0.3dB gain)

• Imperfect relaying
  – Relay perf. versus Dest. perf. (DF protocol)
  – Destination perf.: \textit{BW has approximated perf. as FW}
  – The destination has approximated perf. as the relay.

\textit{Relay performances are crucial to the relay system!}
Conclusions

• The optimization framework for the LDPC-coded cooperative relay systems
• Efficient approaches for code performance analysis
• The optimized codes can achieve significant gains over the regular codes;
• Under the DF protocol, the optimized codes can approach within 0.1dB gap to the capacity;
• Relay performances are crucial to the system performances.