Codes for Node-Constrained Relaying

Gerhard Kramer
Bell Labs, Lucent Technologies
gkr@bell-labs.com
Outline

1) Network nodes
2) Wireline relaying
   - one path: how data compression helps
3) Generalizations and Problems
   - one relay, two paths
   - two relays, two paths
   - many sources/paths, unicast/multicast
1) Network Nodes

n Wireline

Node constraints:
Suppose k ports can be active at once, e.g., k=1

n Wireless

Half-duplex constraint:
\[ y_t = \begin{cases} 
  Z_t + \sum_{s \neq t} \frac{h_{st}}{d_{st}} x_s & \text{if } X_t = 0 \\
  0 & \text{else}
\end{cases} \]
Networks

Wireline

Wireless

\[ y_2 = \begin{cases} 
Z_2 + \frac{h_{12}}{d_{12}^{a/2}} X_1 & \text{if } X_2 = 0 \\
0 & \text{else} 
\end{cases} \]

\[ y_3 = Z_3 + \frac{h_{13}}{d_{13}^{a/2}} X_1 + \frac{h_{23}}{d_{23}^{a/2}} X_2 \]
2) Wireline Relaying

3 node example:

Suppose 1 port/node can be active simultaneously. A link (channel) model:

if $X_2=0$ then $Y_2=X_1$
if $X_2 \neq 0$ then $Y_2=0$

Suppose the random variables are bits*.

*Usually the $X_i$ are packets, and not bits, but the following gives the general idea
Guess: capacity is \( \frac{1}{2} \) bit/use (or packet/use)?

A “decompression” code at node 2:

Node 2 transmits appropriate branch labels upon receiving \( X_1 \).
For example:

\[
X_1 = 0, 1, X0, 0, 1, X1, X1, X0 \\
X_2 = 0, 0, 10, 0, 0, 10, 10, 10, 0
\]

1st network edge: every \( X_2 \) word has one zero

2nd network edge: \( R \leq \frac{1}{E[L_2]} = \frac{2}{3} \) bits/use!
Better compression codes (e.g., Huffman codes, arithmetic source codes) achieve $R = 0.773$ bits/use with $\Pr[X_2=0] = 0.773^*$. 

How can we understand this gain? Is 0.773 the capacity of this network?

*This is when $\Pr[X_2=0] = h(\Pr[X_2=0])$, where $h(x) = -x\log_2 x - (1-x)\log_2(1-x)$ is Shannon's binary entropy function.
Capacity Bounds

- The channel is memoryless and physically degraded so decode-and-forward (DF) achieves capacity (Cover & El Gamal, 1979):

\[ C = \max_{p(x_1,x_2)} \min \left[ I(X_1;Y_2|X_2), I(X_1X_2;Y_3) \right] \]

- Choose \( X_1 \) and \( X_2 \) independent and \( X_1 \) coin-tossing

\[ I(X_1;Y_2|X_2) = H(Y_2|X_2) = \Pr[X_2=0] \]
\[ I(X_1X_2;Y_3) = H(Y_3) = h(\Pr[X_2=0]) \]

- Note: our new DF protocol based on source coding has low-delay and variable-length codewords

- Many relays, one path: capacity via DF (Aref, 1980)
3) Generalizations & Problems

- One relay, **two** paths:

- A memoryless (non-degraded but) **deterministic** relay channel. The capacity is (El Gamal & Aref, 1982)

\[ C = \max_{p(x_1, x_2)} \min \left[ H(Y_2 Y_3 | X_2), H(Y_3) \right] \]

- Strategy: use “partial” DF which is a path-based protocol that superposes two DF schemes.

- Problem: what kind of source coding works here?
Two relays, two paths (a Schein-Gallager network):

Capacity is not known (even with no constraint at $Y_4$)

A path-based strategy (Gupta & Kumar, 2003) achieves $R = R_1 + R_2$ where

\[
R_1 = \min \left[ I(U_2;Y_2|X_2), I(U_2X_2;Y_4) \right]
\]

\[
R_2 = \min \left[ I(U_3;Y_3|X_3), I(U_3X_3;Y_4|X_2U_2) \right]
\]

\[
P(u_2,u_3,x_2,x_3) = P(u_2,x_2)P(u_3,x_3)
\]
Many sources/paths, unicast/multicast:

- Suppose every node has a 1-port constraint
- Basic routing throughput: 1/2 bit/use
- Basic network coding: 2/3 bits/use
- Relay routing: 0.732 bits/use*

*in general, one should combine network coding and relaying
*for packets, the gains are smaller but do permit “covert” communication
Summary

Two Basic Open Problems:

- Find the unicast and multicast capacities of deterministic relay networks.
  (Is the cut-set upper bound achievable or not?)

Partial results:

- No node constraints and no interference:
  - unicast: use routing (Ford & Fulkerson, 1956)

- No node constraints but with broadcasting:
  - multicast: use (modified) network coding (Ratnakar & K, 2005)