Wireless Traffic in a Network with Random Connections

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Well-Known Model of Network of Wireless Devices

- All sharing same frequency
- Pairs talking simultaneously and interfering with one another

\[ \frac{1}{r^m} \]

\( n \) nodes
P. Gupta & friends

- Order $n$ bit-meters of traffic flow possible, but not more (for $m > 2$)
  
  (I will ignore almost all constants)

- The average user is $\sqrt{n}$ meters away, so the traffic flow across the network is $\sqrt{n}$ bits

- Or $1/\sqrt{n}$ bits per user
The Reason: Interference

- Problem: When you transmit you interfere with your neighbors out to a certain distance.

When A wants to talk to B, all nodes in circle around A cannot receive something else and all nodes in circle around B cannot transmit something else.
New Model for Network of Devices

- Independent identically distributed random connections \( h \) whose power \( \gamma = |h|^2 \) is drawn from some density \( p(\gamma) \)
Rationale for Model

Sources of Rayleigh and shadow fading

Link strength dependent more on signal bounces and existence of obstacle than distance

http://mars.bell-labs.com
Some Possible Densities

- $p(\gamma) = e^{-\gamma}$ (Rayleigh fading)
- $p(\gamma) = (1-p_n) \delta(\gamma) + p_n \delta(\gamma-1)$ (shadow fading)
- $p(\gamma) = \frac{4\pi\Delta}{nm} \frac{1}{\gamma^{1+\frac{2}{m}}}$

where: $\Delta$ is the density of nodes (in nodes/meter$^2$)
$m$ is the distance power decay exponent

This density gives the same marginal distribution of powers as the distance law
How Nodes Talk

- Pair requirement:
  - Transmission allowed only between “good” user pairs, where $\gamma > \beta$
  - Hop along relays:

![Diagram of good paths between sources and destinations with interference points marked]

Sources

Destinations

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Minimum SINR Requirement

- SINR requirement:
  \[ \frac{P \gamma_{i_k,j_k}}{\sigma^2 + P \sum_{l \neq k} \gamma_{i_l,j_k}} \geq \rho_0 \]

Total throughput \( T \) is the number of users that can transmit simultaneously multiplied by the data rate \( \log(1+\rho_0) \), divided by the number of hops.

We use results from graph theory, probability theory.
Preview of Results

• Rayleigh fading model allows only $T = \log n$ simultaneous pairs to talk. This is bad!

• Shadow fading model can be much better, although some values of $p_n$ are clearly bad! (for example $p_n = 0$ or $p_n = 1$). For $p_n = (\log n)/n$ the throughput is $T = n \cdot \log^2(\log n)/\log^3 n$ which is great!
Shadow Fading Throughput as Function of $p_n$

Peak for $n=1000$ approximately $(\log n)/n$
Mechanism of Proof

- Communicate along “good” channels using intermediate relays as hops:
  - All channels whose powers exceed some threshold \( \beta \) are good:
    - Too large and we cannot get from a source to a destination
    - Too small and we are not boosting the SINR
- Establish a schedule of relays that do not collide
  - Relays decode and retransmit message
  - No node can transmit and receive simultaneously
- Compute interference generated by each transmission
- Compute the SINR at each relay
- Compute threshold that yields maximum throughput

\[
T = (1 - \varepsilon) \frac{k}{h} \log(1 + \rho_0)
\]
Threshold Induces Random Graph

Graph is established by drawing a line whenever $\gamma$ exceeds $\beta$

The graph is $G(n,p)$ whose edge probability is $p = P(\gamma > \beta)$
Hops Without Collisions — Relays

• (Broder et al. 1996) Suppose that the graph $G(n,p)$ has
  $p \geq \frac{(\log n)}{n}$. Then with probability $\rightarrow 1$ there are vertex-
  disjoint paths connecting $s_i$ to $d_i$ for $i = 1, \ldots k$ provided that
  $k \leq \alpha n (\log np)/\log n$ for some $\alpha > 0$.
  • We can therefore establish up to $\alpha n (\log np)/\log n$ non-colliding
    paths.
  • Every message takes no more than approximately $(\log n)/(\alpha \log np)$
    hops

• However, we still need to ensure that the probability of error
  can be made small
Errors Made by Relays

- \( \varepsilon = P(\text{Message } i \text{ is dropped}) = P(\cup_i \text{ SINR at relay } i \leq \rho_0) \)
  
  \[ \leq \text{(# hops)} \times P(\text{SINR at relay } 1 \leq \rho_0) \]
  
  \[ \leq \frac{\log n}{\alpha \log np} \frac{\sigma_\gamma^2 / (k-1)}{\left( \frac{p \beta - \rho_0 \sigma_\gamma^2}{(k-1)p \rho_0 - \mu_\gamma} \right)^2} \]

- We use Chebyshev bound, which is valid if

  \[ \rho_0 \leq \frac{P \beta}{\sigma_\gamma^2 + P(k-1)\mu_\gamma} \]

- Can always make \( \varepsilon \) go to zero by making \( \rho_0 \) small enough.
Obtaining Main Result

- Fix $\beta$: This fixes $p$ and the number of hops $h$
- Bound number of simultaneous non-colliding pairs using Broder result
- Maximize throughput $T = (1 - \varepsilon) \frac{k}{h} \log(1 + \rho_0)$ subject to $\varepsilon \to 0$ as $n \to \infty$
  - Possible to show that the best $\rho_0$ is
    
    $\rho_0 = \frac{a \beta}{\frac{\sigma^2}{p} + (k(\beta) - 1)\mu_y}$

    for some $0 \leq a \leq 1$
- Now maximize over $\beta$ (this can be hard)
Main Result

- Let $Q(x) = P(\gamma \geq x)$. Choose $\beta$ such that $Q(\beta) = (\log n)/n$. Then the throughput

$$T = (1 - \varepsilon)k(\beta)\alpha \frac{\log(nQ_n(\beta))}{\log n} \log \left( 1 + \frac{a\beta}{\sigma^2 + (k(\beta) - 1)\mu} \right)$$

is achievable where $k(\beta)$ obeys the Broder constraint
### Heavy Dependence on Distribution

<table>
<thead>
<tr>
<th>Distribution</th>
<th>$p(\gamma)$</th>
<th>Throughput</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shadow</td>
<td>$(1 - p_n)\delta(\gamma) + p_n\delta(\gamma - 1)$</td>
<td>$\frac{\log^2(\log n)}{\log^3 n}n$</td>
</tr>
<tr>
<td>Exponential</td>
<td>$e^{-\gamma}$</td>
<td>$\log n$</td>
</tr>
<tr>
<td>(Rayleigh)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Decay</td>
<td>$\frac{4\pi\Delta}{nm} \frac{1}{\gamma^{1+\frac{2}{m}}}$, $m &gt; 2$</td>
<td>$\frac{\log^2(\log n)}{(\log n)^{2 + \frac{m}{2}}}n$</td>
</tr>
<tr>
<td>Heavy Tail</td>
<td>$\frac{c}{1 + \gamma^4}$</td>
<td>$\frac{\log \log n}{\log^{4/3} n}n^{1/3}$</td>
</tr>
</tbody>
</table>

[Link](http://mars.bell-labs.com)
Shadow Fading Throughput as Function of $n$
Rayleigh Fading Throughput as Function of $n$
Extensions

- Upper bounds?
  - So far we have achievability results with only some upper bounds

- What kind and amount of fading is optimum for a network?
  - Shadow-fading results are particularly good, provided a large amount of shadowing is present.
  - Rayleigh results particularly bad

- Do real networks have “good” amounts of fading?
  - Experiments needed!
  - If not, can we artificially induce beneficial effects by passive or active methods?
More Information

Found on MARS web-site:

http://mars.bell-labs.com